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# The Nusselt condensation and nonisothermality

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**Abstract**

We extend Nusselt's theory of condensation accounting for subcooling of condensate at a variable wall temperature. Neglecting vapour shear, we derive an equation for heat transfer of a saturated pure vapour condensing on a surface of arbitrary shape. Using this equation, we examine the interaction between heat transfer and nonisothermality specifying the conditions that give the mean heat transfer coefficient independent of the temperature profiles. Applying these conditions to the condensation on a horizontal circular tube and on a sphere, we analytically confirm some numerical results from the literature. © 1998 Elsevier Science Ltd. All rights reserved.

**Nomenclature**

$A$ area of cooling surface
$B$ parameter, amplitude of temperature
$c$ specific heat capacity
$C$ constant of integration, also integral equation (34)
$D$ tube (sphere) diameter
$f$ nonisothermality function
$F$ function depending on surface shape
$g$ acceleration due to gravity
$\Delta h$ latent heat
$\mathfrak{J}$ extension of cooling surface orthogonal to condensate flow
$k$ thermal conductivity
$Ku$ Kutateladze number
$P$ pressure
$Q$ heat flow
$q$ heat flux
$s$ coordinate in direction of condensate flow, arc length, Fig.1
$S$ arc length of cooling surface in direction of condensate flow.

*Greek symbols*

$\alpha$ local heat transfer coefficient
$\bar{\alpha}$ mean heat transfer coefficient
$\chi$ nondimensional quantity, equation (13)
$\delta$ local thickness of condensate film
$\phi$ angle, Fig. 1
$\nu$ kinematic viscosity
$\Delta\rho$ density difference, $\Delta\rho = \rho_L - \rho_V$
$\vartheta$ temperature
$\Delta\vartheta$ average temperature difference
$\tau$ shear stress.

*Subscripts*

I interface, saturation
L liquid
$s$ in $s$ direction
W wall, cooling surface
0 constant value.

## 1. Introduction

In an article, published in this journal, Memory and Rose [1] treated free convection laminar film condensation on a horizontal circular tube, the temperature of which was assumed to be constant in the axial, but to obey a cosine distribution in the circumferential direction. Velocity and temperature profiles in the film were those of the Nusselt [2] model. The differential equation for heat transfer thus obtained was solved numerically, demonstrating that the mean heat transfer coefficient is practically independent of the amplitude of the circumferential temperature distribution. We quote from the text following equation (17) of [1]: "In fact $C$ increased slightly with increasing $A$ but was constant to

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four significant figures, with a value of 0.7280 for all values of $A$." For reasons of completeness, we should add to this statement that $C$ represents an integral, the integrand of which contains the function of the temperature distribution with $A$ as its amplitude. Recently, Hsu and Yang [3] arrived at a similar conclusion concerning free convection condensation on a sphere.

Condensation on a surface of variable temperature has already been studied by Bromley et al. [4]. They also measured temperature distribution on the circumference of a horizontal circular tube, showing the mean heat transfer coefficient, determined with the average temperature difference, to largely obey the Nusselt theory. Neglecting subcooling of condensate, Labuntsov [5] published an analysis of free convection condensation under nonisothermal conditions which is seemingly the most general so far reported. Zhou and Rose [6] examined the role played by both heat conduction and convection in the angular direction in a liquid film established by condensation of a saturated vapour on a horizontal circular tube in the free convection region. They showed that, for the adopted distribution of the surface temperature, the contributions by these transport mechanisms to heat transfer can be neglected. For further sources on this subject, the reader may be referred to the article by Hsu and Yang [3] and to a very recent review paper by Rose [7]. An overview, provided by Fujii [8], contains several experimental findings and may particularly be recommended.

In a preceding paper [9], we generalised the Nusselt theory. In an Appendix to that paper, we also gave basic equations for the case of a variable wall temperature and briefly discussed some results from the literature. In the present paper, we continue and extend the considerations sketched in [9]. Our central aim is now to state more generally conditions for the mean heat transfer coefficient, formed with the average temperature difference, to be independent of distributions of the temperatures forcing condensation. Having established the conditions, we apply them, as examples, to the condensation cases numerically treated by Memory and Rose [1] and by Hsu and Yang [3]. Performing integration of their expressions describing free convection condensation, we confirm their numerical results, showing that the amplitude of the temperature distribution adopted by the authors does, indeed, not affect the mean heat transfer coefficient.

## 2. The governing differential equations and their solutions

### 2.1. *Expression for the heat flux*

Using the velocity and temperature profiles in the condensate film as in Nusselt's theory, one obtains on the basis of conservation laws, see e.g. [9], the following equation for the local heat flux $q_W$:

$$q_W = \frac{1}{3}\frac{\Delta h \Delta\rho}{\nu_L}\frac{1}{\Im}\frac{d}{ds}\left(\left(1+\frac{3}{8}\frac{c_{pL}(\vartheta_I-\vartheta_W)}{\Delta h}\right)\Im g_s \delta^3 + \frac{3}{2}\left(1+\frac{1}{3}\frac{c_{pL}(\vartheta_I-\vartheta_W)}{\Delta h}\right)\frac{\tau_I \Im \delta^2}{\Delta\rho}\right) \quad (1)$$

with

$$g_s = F(s)_g. \quad (2)$$

The symbols $g$, $\Delta\rho$, $c_{pL}$, $\Delta h$ and $\nu_L$ have the common meanings; $\Im$ represents the extension of the cooling surface orthogonal to the direction of condensate flow; $\delta$ is the film thickness, $\vartheta_I$ and $\vartheta_W$ are the temperatures, Fig. 1. The function $F(s)$, where $s$ is the arc length, describes the action of gravity on condensate flow. For a vertical surface, $F(s) = 1$; for an inclined plate, a sphere, and a horizontal circular tube it is $F(s) \equiv F(\phi) = \sin\phi$.

Equation (1) is generally valid within the Nusselt model; it takes physical properties as constant, but allows changes of all other quantities, including also the wall temperature $\vartheta_W$, along the flow path $s$. The heat flux $q_W$ is therefore constant over the film thickness. It consists of two terms. The first term arises from the action of the modified gravity $g_s$ on condensate flow, the second term is associated with the shear stress $\tau_I$. The contributions

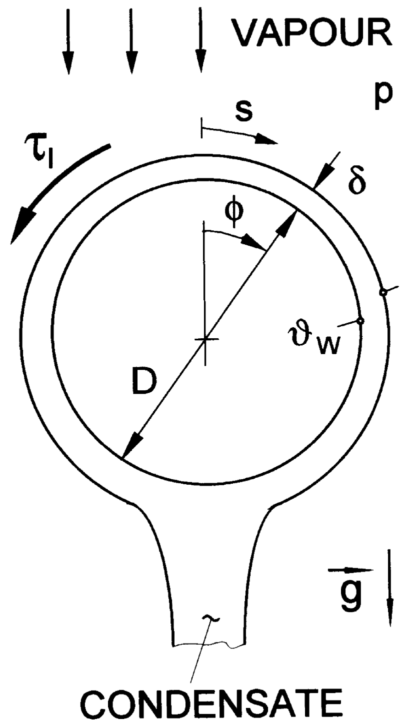


Fig. 1. Condensation of saturated vapour illustrated for a sphere and a horizontal tube.

of these terms to the heat flux are different and depend on both the thermodynamic state of the system and condensation conditions. They are, in general, not independent of each other; the gravity and shear stress terms interact at least over the temperature difference $\vartheta_I - \vartheta_W$. For example, a higher temperature difference may result in a larger flux of condensing vapour on the film surface, thereby influencing the shear stress.

The temperature difference $\vartheta_I - \vartheta_W$, appearing in the two terms of equation (1), accounts for the condensate subcooling. The subcooling effect on heat transfer can mostly be neglected at a low saturation temperature, but not in the region near the thermodynamic critical point, where $c_{pL}$ increases and $\Delta h$ decreases as the critical temperature is approached.

In the present paper, we neglect the vapour shear at the film surface, that is, we set $\tau_I = 0$. This condition is known from the literature to specify free convection condensation implying that the vapour velocity be the same everywhere and, at the same time, coincides with that of the film surface. Strictly viewed, this is impossible in a general case of a varying film thickness $\delta$ and/or a changing function $F(s)$ in equation (2) in the direction of condensate flow.

### 2.2. The film thickness and the heat transfer coefficient

The heat flux $q_W$ is used to define a local heat transfer coefficient $\alpha$,

$$q_W = \alpha(\vartheta_I - \vartheta_W) = \frac{k_L}{\delta}(\vartheta_I - \vartheta_W), \tag{3}$$

where $k_L$ is the thermal conductivity of condensate.

If the temperature difference $\vartheta_I - \vartheta_W$ varies, we may, following Labuntsov [5], see also Memory and Rose [1] and Rose [7], introduce an average temperature difference $\Delta\vartheta$ by

$$\vartheta_I - \vartheta_W = \Delta\vartheta f(s) \tag{4}$$

with $f(s)$ as a temperature distribution function accounting for the nonisothermality effect in the direction of condensate flow. By definition,

$$\frac{1}{A}\int_A f(s)\,\mathrm{d}A \equiv 1, \tag{5}$$

if $A$ is the area of the cooling surface and $\mathrm{d}A$ its element. Therefore, for a constant temperature difference $\vartheta_I - \vartheta_W$, it is $f(s) = 1$.

Setting $\tau_I = 0$ in equation (1) and combining with equations (3) and (4), gives

$$\frac{k_L}{\delta}\Delta\vartheta f(s) = \frac{1}{3}\frac{\Delta h \Delta\rho g}{\nu_L}\frac{1}{\mathfrak{J}}\frac{\mathrm{d}}{\mathrm{d}s}\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{J}\delta^3 F(s)\right), \tag{6}$$

where the Kutateladze number $Ku$,

$$Ku = \frac{c_{pL}\Delta\vartheta}{\Delta h}, \tag{7}$$

measures the maximum subcooling of condensate in terms of the latent heat. This quantity is sometimes called the number of phase change or the Jakob number.

When integrated, equation (6) delivers the film thickness $\delta$,

$$\delta = \frac{(2/D)^{1/4}\delta_0}{\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{J}F(s)\right)^{1/3}} \times \left(\frac{4}{3}\int_0^s \left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{J}F(s)\right)^{1/3}\mathfrak{J}f(s)\,\mathrm{d}s + C\right)^{1/4}, \tag{8}$$

with $C$ as an integration constant, and

$$\delta_0 = \left(\frac{3}{2}\frac{\nu_L k_L \Delta\vartheta D}{\Delta\rho\Delta h g}\right)^{1/4}. \tag{9}$$

The arbitrary length $D$ (more precisely $D/2$) introduced here simplifies the matter in that the quantity $\delta_0$ takes the nature of a length. With condensation on a horizontal circular tube or on a sphere of the diameter $D$, $\delta_0$ is shown later to become equal or proportional (for sphere) to the film thickness at $s = 0$ ($\phi = 0$), if there is no condensate inundation, Fig. 1.

The constant $C$ of integration in equation (8) depends on the shape of the surface. For condensation on a vertical bank of horizontal tubes and on a hanging chain of spheres, the constant $C$ was determined in [9] assuming the driving temperature difference to be constant, that is, for $f(s) = 1$. The same procedure applies also for $f(s) \neq 1$. In the case that $C = 0$, $Ku = 0$ and $\mathfrak{J} = \text{const}$, equation (8) simplifies to an expression first derived by Labuntsov [5].

The mean heat transfer coefficient $\bar{\alpha}$ over an area $A$ of the surface is defined by

$$\dot{Q} = \bar{\alpha}\Delta\vartheta A = \int_A q_W\,\mathrm{d}A,$$

from which, considering equation (3), further, putting $\mathrm{d}A = \mathfrak{J}\,\mathrm{d}s$, we obtain

$$\bar{\alpha} = \frac{1}{A}\int_A \alpha\mathfrak{J}\,\mathrm{d}s = \frac{k_L}{A}\int_A \frac{\mathfrak{J}}{\delta}\,\mathrm{d}s. \tag{10}$$

Combining expressions (8) and (10) gives

$$\bar{\alpha} = \frac{k_L}{\delta_0}\left(\frac{D}{2}\right)^{1/4}\frac{1}{A} \times \int_0^s \frac{\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{J}F(s)\right)^{1/3}\mathfrak{J}f(s)\,\mathrm{d}s}{\left(\frac{4}{3}\int_0^s\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{J}F(s)\right)^{1/3}\mathfrak{J}f(s)\,\mathrm{d}s + C\right)^{1/4}} \tag{11}$$

or, after integration,

$$\bar{\alpha} = \frac{k_L}{\delta_0}\chi = \left(\frac{2}{3}\frac{\Delta\rho\Delta h g k_L^3}{\nu_L \Delta\vartheta D}\right)^{1/4}\chi, \tag{12}$$

where

$$\chi = \left(\frac{D}{2}\right)^{1/4}\frac{1}{A}\left(\left(\frac{4}{3}\int_0^S\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{I}F(s)\right)^{1/3} \times \mathfrak{I}f(s)\,\mathrm{d}s + C\right)^{3/4} - C^{3/4}\right). \tag{13}$$

Depending on the functions $\mathfrak{I}$, $F(s)$ and $f(s)$, and on condensation conditions, the integral in equation (13) can be solved analytically, written more compactly, or must be evaluated numerically in this form. Equation (12) is generally valid at $\tau_I = 0$, see Fig. 1.

For simplicity, we assume in the following a vapour condensation without condensate inundation. Then, $C = 0$, and, from equation (13)

$$\chi = \left(\frac{D}{2}\right)^{1/4}\frac{1}{A}\left(\frac{4}{3}\int_0^S\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{I}F(s)\right)^{1/3}\mathfrak{I}f(s)\,\mathrm{d}s\right)^{3/4}. \tag{14}$$

This equation makes the basis of our further analysis, where we first apply it to a few familiar condensation cases at a constant temperature difference, and then discuss the interaction between heat transfer and non-isothermality. In this connection, we may note that the analysis, undertaken below, is not confined to condensation without condensate inundation ($C = 0$); it is also applicable to condensation cases with condensate inundation, for which $C \neq 0$. A way of obtaining the constant $C$ is described in a previous study [9].

## 3. Constant temperature difference

For a constant temperature difference, $f(s) = 1$, equation (14) reduces to

$$\chi = \left(1+\frac{3}{8}Ku\right)^{1/4}\left(\frac{D}{2}\right)^{1/4}\frac{1}{A}\left(\frac{4}{3}\int_0^S(\mathfrak{I}F(s))^{1/3}\mathfrak{I}\;\mathrm{d}s\right)^{3/4}. \tag{15}$$

This equation, combined with equation (12), contains all the cases of the Nusselt condensation of a saturated vapour. For example, if condensation is taking place on a vertical tube of a constant diameter $D$, or on a vertical plate, we have $\mathfrak{I} = $ const. and $F(s) = 1$. Then,

$$\chi = \frac{4}{3}\left(1+\frac{3}{8}Ku\right)^{1/4}\left(\frac{1}{4}\frac{3}{2}\frac{D}{S}\right)^{1/4}, \tag{16}$$

where $S$ is the tube length, or the plate height. Taken together, equations (12) and (16) result in the original Nusselt expression, if $Ku = 0$.

In the case of a horizontal circular tube, $\mathfrak{I} = $ const., $\mathrm{d}s = (D/2)\,\mathrm{d}\phi$ and $F(s) = \sin\phi$, Fig. 1. Thus,

$$\chi = \frac{1}{\pi}\left(1+\frac{3}{8}Ku\right)^{1/4}\left(\frac{4}{3}\int_0^\pi \sin^{1/3}\phi\,\mathrm{d}\phi\right)^{3/4} \tag{17}$$

with the numerical value

$$\int_0^\pi \sin^{1/3}\phi\,\mathrm{d}\phi \approx 2.5871.$$

Combining (12) and (17) gives the Nusselt expression for a single tube with the numerical value of the constant 0.7280. The actual value is somewhat larger, which is without any practical significance, see e.g. [7].

Where the cooling surface is a sphere ($F(s) = \sin\phi$, $\mathfrak{I} = D\pi\sin\phi$), equation (15) becomes

$$\chi = \frac{1}{2}\left(1+\frac{3}{8}Ku\right)^{1/4}\left(\frac{4}{3}\int_0^\pi \sin^{5/3}\phi\,\mathrm{d}\phi\right)^{3/4} \tag{18}$$

with

$$\int_0^\pi \sin^{5/3}\phi\,\mathrm{d}\phi \approx 1.68262.$$

Equations (12) and (18) result in an expression derived first by Dhir and Lienhard [10]. The numerical value of the constant in the expression for the mean heat transfer coefficient obtained by these authors is 0.785; it disagrees with the corresponding value obtained in the present paper. A similar situation occurs with the results by Yang [11], who reported the constant to be 0.803. However, our numerical value of 0.8282 agrees with that by Hsu and Yang [3]. Note that also here, like for tube, the actual value of the constant is insignificantly larger.

## 4. Variable temperature difference

### 4.1. *Independence of heat transfer coefficient from non-isothermality function*

In this section, we examine the conditions for the mean heat transfer coefficient $\bar{\alpha}$ to be independent of the temperature distribution function $f(s) \equiv f$. Such conditions require the relation

$$\frac{\partial\chi}{\partial f} = 0, \tag{19}$$

which, with $\chi$ from equation (14), gives the expression

$$\frac{\partial}{\partial f}\int_0^S\left(\left(1+\frac{3}{8}Kuf(s)\right)\mathfrak{I}F(s)\right)^{1/3}\mathfrak{I}f(s)\,\mathrm{d}s = 0. \tag{20}$$

A further treatment of equation (20) seems impossible in the general case. Thus, assumptions about the members of the integrand are needed and, as an example, we force

$$\left(1+\frac{3}{8}Kuf(s)\right)\Im F(s) = \text{const.} \tag{21}$$

over the entire surface. Then, equation (20), divided by $A \neq A(f)$, simplifies to

$$\frac{\partial}{\partial f}\left(\frac{1}{A}\int_A f(s)\,\mathrm{d}A\right) = 0, \tag{22}$$

where $\mathrm{d}A = \Im\,\mathrm{d}s$.

Because of the identity (5), equation (22) is satisfied, whatever the shape of the nonisothermality function $f(s)$. Therefore, in this case, expression (21) assumes the nature of a criterion equation. It connects the nonisothermality function $f(s)$ with the shape of the surface, resulting in the mean heat transfer coefficient $\bar{\alpha}$ independent of $f(s)$.

On the basis of equation (21), we may distinguish two particular cases. One of them is specified by taking the product $\Im F(s)$ as constant, resulting in an invariable function $f(s)$. Clearly, this has no physical significance, except for the isothermal conditions. The other case, namely, $Ku \to 0$, allows some interesting conclusions.

#### 4.1.1. *Case of a small* Ku

For a negligibly small value of $Ku$, the effect of $f(s)$ disappears and equation (21) reduces further, giving

$$\Im F(s) = \text{const.} \tag{23}$$

By inserting this condition into equation (14), and considering equation (5), we obtain

$$\chi = \frac{4}{3}\left(\frac{1}{4}\frac{3}{2}\frac{D\Im F}{A}\right)^{1/4}\left(\frac{1}{A}\int_0^S f(s)\Im\,\mathrm{d}s\right)^{3/4} = \frac{4}{3}\left(\frac{1}{4}\frac{3}{2}\frac{DF}{S}\right)^{1/4}, \tag{24}$$

where $S = A/\Im$ (if $\Im$ varies with $s$, then $\Im \equiv (\int_0^S \Im\,\mathrm{d}s)/S$ as a mean value).

Equation (24) expresses a statement by Labuntsov [5]. It shows that the mean heat transfer coefficient $\bar{\alpha}$, defined with the average temperature difference, is independent of the shape of the nonisothermality of the surfaces, sandwiching the condensate film. The simplest cases, where the equation applies, are those of a plane wall and a vertical tube. However, equation (23) allows creation of a family of surfaces, for which the nonisothermality effect is negligible and equation (24) holds.

#### 4.1.2. *Nonisothermality function depends on a parameter*

If the nonisothermality function contains a parameter, say $B$, so that $f(s) \equiv f(B,s)$, then, with $\partial/\partial f = (\partial/\partial B)(\partial B/\partial f)$, equation (20) may be written as

$$\frac{\partial}{\partial B}\int_0^S \left(\left(1+\frac{3}{8}Kuf(B,s)\right)\Im F(s)\right)^{1/3}\Im f(B,s)\,\mathrm{d}s = 0, \tag{25}$$

for $\partial B/\partial f \neq 0$, or

$$\int_0^S \frac{1+\frac{1}{2}Kuf(B,s)}{\left(1+\frac{3}{8}Kuf(B,s)\right)^{2/3}}\frac{\partial f(B,s)}{\partial B}(\Im F(s))^{1/3}\Im\,\mathrm{d}s = 0, \tag{26}$$

if the integrand in equation (25) is a linear function of $B$. This expression specifies the conditions to be fulfilled for the mean heat transfer coefficient not to depend on $B$. For a very low condensate subcooling, that is, for $Kuf(B,s) \to 0$, or generally, for

$$\frac{1+\frac{1}{2}Kuf(B,s)}{\left(1+\frac{3}{8}Kuf(B,s)\right)^{2/3}} \approx \text{const.}$$

equation (26) reduces to

$$\int_0^S \frac{\partial f(B,s)}{\partial B}(\Im F(s))^{1/3}\Im\,\mathrm{d}s = 0. \tag{27}$$

As an example, equation (27) may now be applied to a horizontal circular tube and to a sphere, where $\mathrm{d}s = (D/2)\,\mathrm{d}\phi$ and $F(s) \equiv F(\phi) = \sin\phi$, for both surfaces. Since for a tube, $\Im = \text{const.}$ and, replacing $s$ and $S$ through $\phi$ and $\pi$, we have

$$\int_0^\pi \frac{\partial f(B,\phi)}{\partial B}\sin^{1/3}\phi\,\mathrm{d}\phi = 0. \tag{28}$$

For a sphere, $\Im = \pi D\sin\phi$, thus,

$$\int_0^\pi \frac{\partial f(B,\phi)}{\partial B}\sin^{5/3}\phi\,\mathrm{d}\phi = 0. \tag{29}$$

To evaluate the integrals in equations (28) and (29), we choose

$$f(B,\phi) = 1 - B\cos\phi, \tag{30}$$

as adopted by Memory and Rose [1] for a horizontal circular tube and by Hsu and Yang [3] for a sphere. The parameter $B$ takes the role of the amplitude of the nonisothermality function.[1] Note that Fuji [8] discusses a theoretical expression for the distribution of the surface temperature at a constant heat flux. His expression, however, is scarcely applicable in the region of condensate departure from the tube; for $\phi = \pi$, it delivers infinite temperature. Therefore, it should not be pursued further in the current article.

With the nonisothermality function (30), the integrals

[1] Condensation experiments by the present author [12] on a horizontal tube having a capillary structure on its outside surface does not confirm the simple shape of equation (30). The wall temperature is generally not symmetrical about $\phi = \pi/2$, as expressed by this equation. The position of the inflection point of the circumferential temperature distribution depends on cooling conditions.

(28) and (29) become zero, showing that the mean heat transfer coefficient does not depend on $B$. In this connection, we should emphasise that the results we arrive at and the conclusion drawn hold under specific conditions only, for example, for $Kuf(B, s) \to 0$, which means for no condensate subcooling or for a very weak variation of $f(B, s)$ over the cooling surface. In a more general case, however, equation (25) and not equation (27) should be used which might give a completely different picture.

The conclusion drawn on the basis of equations (28–30) could be verified by integrating equation (14), which would be possible analytically for a horizontal tube and a sphere at $Ku = 0$, using equation (30) for the function $f(B, s)$. Instead of equation (14), however, we should prefer to take expressions from the literature.

### 4.2. Expressions from the literature

Hsu and Yang [3] obtained for free convection condensation of a pure vapour on a single sphere (expression (31) in their paper) the equation

$$\overline{Nu}\left(\frac{Ja}{Ra}\right)^{1/4} = \frac{1}{2}\int_0^{\pi} \frac{(1-B\cos\phi)\sin^{5/3}\phi\,\mathrm{d}\phi}{\left(2\int_0^{\pi}(1-B\sin^{5/3}\phi\,\mathrm{d}\phi\right)^{1/4}}. \tag{31}$$

For definitions of the quantities $\overline{Nu}$, $Ja$ and $Ra$, the reader may be referred to the original paper. The symbol $B$ in this equation represents the amplitude of the non-isothermality function $f(B, \phi)$, which coincides with those of Memory and Rose [1], the above equation (30). The authors [3] treated equation (31) numerically, concluding that the heat transfer is practically unaffected by $B$.

However, when integrated analytically, equation (31) gives

$$\overline{Nu}\left(\frac{Ja}{Ra}\right)^{1/4} = \frac{1}{3}\left(2\int_0^{\pi}(1-B\cos\phi)\sin^{5/3}\phi\,\mathrm{d}\phi\right)^{3/4}$$
$$= \frac{1}{3}\left(2\int_0^{\pi}\sin^{5/3}\phi\,\mathrm{d}\phi\right)^{3/4} \tag{32}$$

which is independent of $B$.

This result disagrees with our equation (14), which, regarding equation (30), becomes

$$\chi = \frac{1}{2}\left(\frac{4}{3}\int_0^{\pi}\left(1+\frac{3}{8}Ku(1-B\cos\phi)\right)^{1/3} \times (1-B\cos\phi)\sin^{5/3}\phi\,\mathrm{d}\phi\right)^{1/4}. \tag{33}$$

To decide with certainty whether or not $\chi$ depends on $B$, a numerical evaluation of the integral would be necessary. But, even without integration, the complexity of the integrand suggests a dependence of $\chi$ on $B$, except for $Ku = 0$. The disagreement between equation (33) and equation (32), concerning the quantity $B$, originates in the fact that Hsu and Yang [3] allowed $f(s)$ to change in the driving temperature difference, used to define the heat transfer coefficient, but not in the condensate subcooling term, where they put, inconsistently, $f(s) = 1$, see [9].

Further, the integral in the expression for $C$,[2] given by Memory and Rose [1], and quoted in the introduction of the current paper, can easily be performed, resulting in

$$C = \frac{1}{\pi}\int_0^{\pi}\left(\frac{(1-B\cos\phi)\sin^{1/3}\phi\,\mathrm{d}\phi}{\left(2\int_0^{\phi}\sin^{1/3}\phi\,\mathrm{d}\phi-\frac{3}{2}B\sin^{4/3}\phi\right)^{1/4}}\right)$$
$$= \frac{2}{3\pi}\left(2\int_0^{\pi}\sin^{1/3}\phi\,\mathrm{d}\phi\right)^{3/4}, \tag{34}$$

which is independent of the amplitude of the wall temperature variation, and which immediately leads to the Nusselt expression under isothermal conditions.

Equation (34) explains the findings of Bromley et al. [4], who showed the average heat transfer coefficient on horizontal tubes, determined with the average temperature difference, to obey the Nusselt theory, despite the considerable nonisothermality of the cooling surface measured by the authors. The same applies to the experiments by Memory and Rose [1]. In connection with the numerical results reported in [1, 3], we may state that our analytical expressions confirm the high accuracy of the numerical evaluations of the integrals.

## 5. Conclusions

In this work, we derived a general equation for heat transfer in laminar film condensation of a pure, saturated vapour with free convection. The equation accounts for the condensate subcooling and nonisothermality, the latter arising from the wall temperature. The model used is that of Nusselt [2], the shape of the cooling surface is arbitrary.

The general equation is applied to condensation with no condensate inundation and analysed with respect to the shape of nonisothermality. The analysis specifies the conditions to be fulfilled for the mean heat transfer coefficient, obtained with the average temperature difference, to be independent of temperature variations of the surfaces sandwiching the condensate film. In cases of condensation on a horizontal circular tube and on a sphere at no condensate subcooling, it is shown that the shape of the nonisothermality functions used [1, 3] do not affect the condensation heat transfer.

[2] To prevent confusion, we should emphasise that $C$ in equation (34) is completely different from the integration constant $C$ in the above derivations.

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**References**

[1] S.B. Memory, J.W. Rose. Free convection film condensation on a horizontal tube with variable wall temperature. International Journal of Heat and Mass Transfer 34 (1991) 2775–2778.
[2] W. Nusselt. Die Oberflächenkondensation des Wasserdampfes. Zeitschrift VD1 60 (1916) 541–546; 569–575.
[3] C.-H. Hsu, S.-A. Yang. Mixed convection film condensation from downward flowing vapours onto a sphere with variable wall temperature. International Journal of Heat and Mass Transfer 33 (1997) 85–91.
[4] L.A. Bromley, R.S. Brodkey, N. Fishmen. Heat transfer in condensation. Industrial and Engineering Chemistry 44 (1952) 2962–2969.
[5] D.A. Labuntsov. Generalisation of the Nusselt theory of condensation on a surface with a nonuniform temperature distribution. Trudy MEI 63 (1965) 79–84.
[6] Y.-Q. Zhou, J.W. Rose. Effect of two-dimensional conduction in the condensate film on laminar film condensation on a horizontal tube with variable wall temperature. International Journal of Heat and Mass Transfer 39 (1996) 3187–3191.
[7] J.W. Rose. Condensation heat transfer fundamentals. Transactions IChemE Part A (1998) 143–152.
[8] T. Fujii. Overlooked factors and unsolved problems in experimental research on condensation heat transfer. Experimental Thermal and Fluid Science 5 (1992) 652–663.
[9] J. Mitrovic. Towards the generalising of the Nusselt condensation theory. Heat and Mass Transfer (submitted).
[10] V. Dhir, J.H. Lienhard. Laminar film condensation on plane and axisymmetric bodies in nonuniform gravity. ASME Journal of Heat Transfer 93 (1971) 545–547.
[11] J.W. Yang. Laminar film condensation on a sphere. ASME Journal of Heat Transfer 95 (1973) 174–178.
[12] J. Mitrovic. Condensation of pure refrigerants R12, R134a and their mixtures on a horizontal tube with capillary structure—an experimental study. Forschung im Ingenieurwesen (1998) (in press).